

## EMEC-5173: Intelligent Tools for Engineering Applications

### Assignment #1

Due time: 5:30PM, Monday, Jan. 29

#### Question 1.1 (Book 1)

- If “ $A \rightarrow B$ ” is  $F$  and “ $B \rightarrow A$ ” is  $T$ , what is the truth value of “ $(A \rightarrow B)$  AND  $(B \rightarrow A)$ ”? Using this result and the truth table of “ $A \rightarrow B$ ”, determine the truth table of “ $A \leftrightarrow B$ ”.

#### Question 2.7

Using common knowledge, experience, judgment, and perception, construct and sketch appropriate membership functions for the following sets:

- |                            |  |
|----------------------------|--|
| (i) Tall men               | (v) Hot outside temperature                |
| (ii) Tall women            | (vi) Cold outside temperature              |
| (iii) Hot room temperature | (vii) Fast car speed on interstate highway |
| (iv) Cold room temperature | (viii) Fast car speed in city              |

In each case, you must give either the functional relation for the membership function, with appropriate numerical values for their parameters, or numerical data to completely represent the membership function.

#### Question 2.8

Two fuzzy sets  $A$  and  $B$  are represented by the following two membership functions:

$$\begin{aligned}\mu_A(x) &= \max\left(0, \frac{x-3}{7}\right) \quad \text{for } x \leq 10 \\ &= \max\left(0, \frac{17-x}{7}\right) \quad \text{for } x > 10\end{aligned}$$

$$\begin{aligned}\mu_B(x) &= \max\left(0, \frac{x-8}{2}\right) \quad \text{for } x \leq 10 \\ &= \max\left(0, \frac{12-x}{2}\right) \quad \text{for } x > 10\end{aligned}$$

- Sketch these membership functions.
- What do  $A$  and  $B$  approximately represent?
- Which one of the two sets is fuzzier?

#### Question 2.9

Consider a fuzzy set  $A$  in the universe  $\mathfrak{R}$  (i.e., the real line) whose membership function is given by

$$\begin{aligned}\mu_A(x) &= 1 - |x - 2| \quad \text{for } |x - 2| \leq 1 \\ &= 0 \quad \text{otherwise}\end{aligned}$$

- (a) Sketch the membership function.
- (b) What is the support set of  $A$ ?
- (c) What is the  $\alpha$ -cut of  $A$  for  $\alpha = 0.5$ ?

**Question 2.11**

The characteristic function  $\chi_A$  of a crisp set  $A$  is analogous to the membership function of a fuzzy set, and is defined as follows:

$$\chi_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases}$$

Show that

$$\begin{aligned} \chi_{A'} &= 1 - \chi_A \\ \chi_{A \cup B} &= \max(\chi_A, \chi_B) \\ \chi_{A \cap B} &= \min(\chi_A, \chi_B) \\ \chi_{A \rightarrow B}(x, y) &= \min[1, \{1 - \chi_A(x) + \chi_B(y)\}] \end{aligned}$$

where  $A$  and  $B$  are defined in the same universe  $X$ , except in the last case (implication) where  $A$  and  $B$  may be defined in two different universes  $X$  and  $Y$ .

What are the implications of these results?

**Question 2.17**

In the usual notation, the T-norm of two membership functions  $x$  and  $y$  is denoted by  $xTy$  and its complementary (or conjugate) norm, the S-norm, is denoted by  $xSy$ . The two norms are related through the DeMorgan's Law

$$xSy = 1 - (1 - x)T(1 - y)$$

Using this relationship, determine the S-norm corresponding to each of the following two T-norms:

- (i)  $AB$
- (ii)  $\max[0, A+B-1]$

Clearly indicate all the important steps of your derivations.

In operations with fuzzy sets, if T-norm represents a generalized intersection, what does S-norm generally represent from this question?