EMEC-5173: Intelligent Tools for Engineering Applications

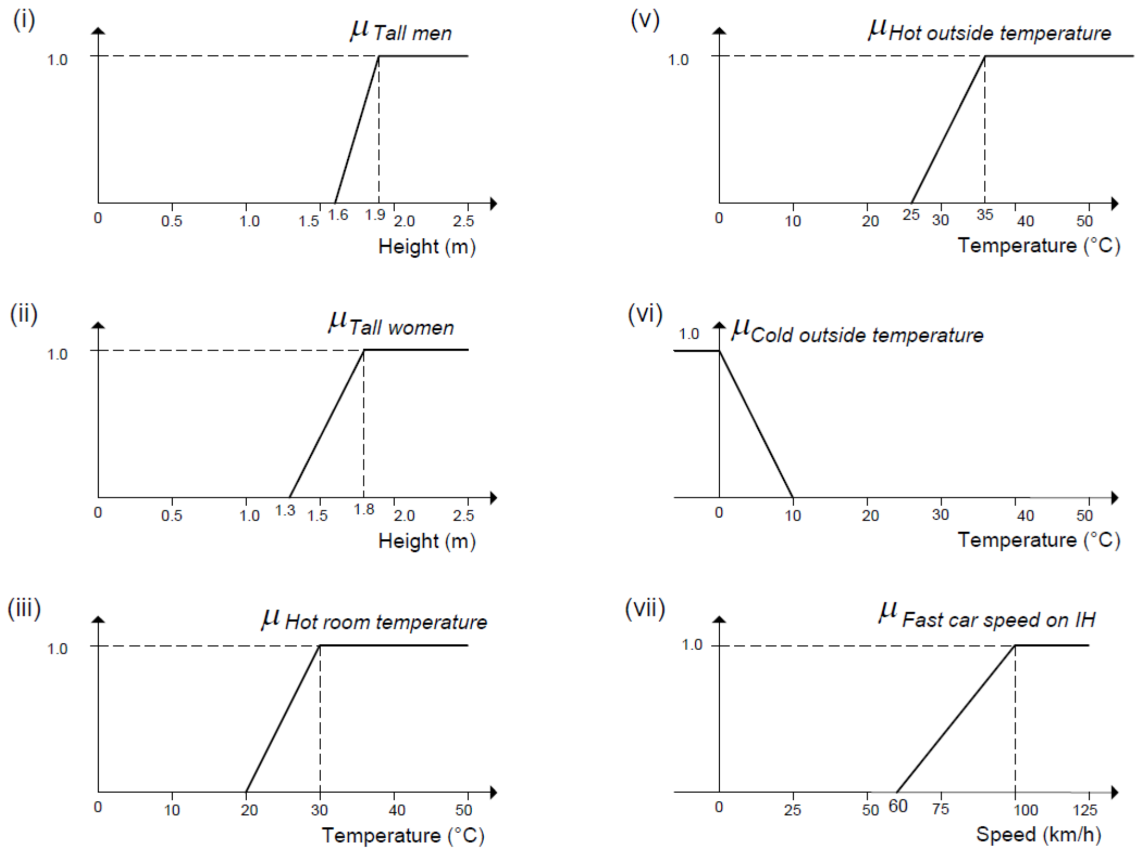
Assignment #1 Reference Solution

Q 1.1

1.1 *F*

A	*B*	*A*↔*B*
T	*T*	*T*
F	*F*	*F*
F	*T*	*F*
F	*F*	*T*

Q 2.7



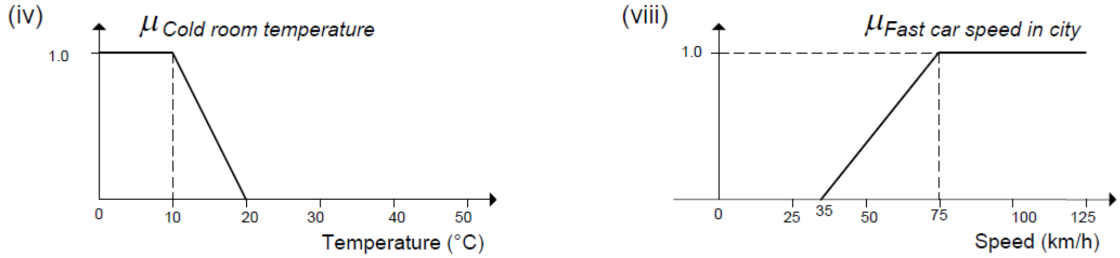


Figure S2.7 Possible membership functions for various fuzzy sets.

Note that these membership functions can vary depending on common knowledge, experience, judgment, and perception of the individual who proposes them.

Q 2.8

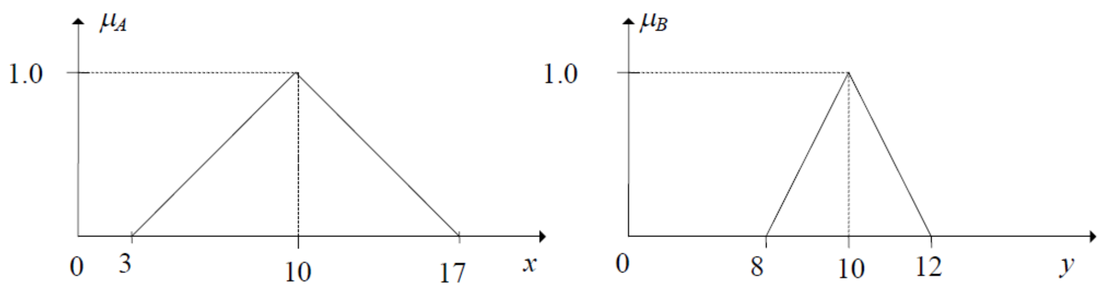
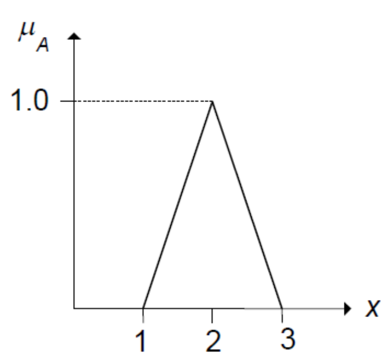


Figure S2.8 The two membership functions.

(b) Both A and B approximately represent the number 10.

(c) Since $\mu_A(x)$ is broader than $\mu_B(x)$ it follows that A is fuzzier than B.

Q 2.9



(b) Support set is $1 \le x \le 3$

(c) For $\alpha = 0.5$, A_α is given by $1.5 \le x \le 2.5$.

Q 2.11

The results follow directly from the corresponding truth tables (strictly, "membership tables), in view of the isomorphism of crisp sets and binary logic, knowing that "T" corresponds to the membership value 1 and "F" corresponds to the value "0".

(a) To show that $\chi_{A'} = 1 - \chi_A$

Membership Table

χ_A	$\chi_{A'}$	$1 - \chi_A$
1	0	1-1=0
0	1	1-0=1

We see that the last two columns are identical, which proves the given relation for the characteristic function.

(b) To show that $\chi_{A \cup B} = \max(\chi_A, \chi_B)$

Membership Table

χ_A	χ_B	$\chi_{A \cup B}$	$\max(\chi_A, \chi_B)$
1	1	1	1
1	0	1	1
0	1	1	1
0	0	0	0

(c) To show that $\chi_{A \cap B} = \min(\chi_A, \chi_B)$

Membership Table

χ_A	χ_B	$\chi_{A \cap B}$	$\min(\chi_A, \chi_B)$
1	1	1	1
1	0	0	0
0	1	0	0
0	0	0	0

(d) Show that $\chi_{A \to B}(x, y) = \min[1, \{1 - \chi_A(x) + \chi_B(y)\}]$

Membership Table

χ_A	χ_B	$\chi_{A \to B}$	$\min[1, \{1 - \chi_A(x) + \chi_B(y)\}]$	$\min(\chi_A, \chi_B)$
1	1	1	1	1
1	0	0	0	0
0	1	1	1	0
0	0	1	1	0

In this last result, 2nd and 3rd columns are identical, thus proving the given relation.
NOTE: $\max\left[\{1-\chi_A(x)\}, \{\chi_B(y)\}\right]$ will also give the membership result as in columns 3 and 4, confirming the "NOT *A* OR *B*" also corresponds to "*A*→*B*."

These four results justify our use of "1-(.)," "max," and "min" operations for *complement*, *union*, and *intersection*. But the use of "min" for *fuzzy implication* is not justified in view of the crisp result given in part (d), because the columns 4 and 5 in the corresponding table are not identical.

Q 2.17

(i) $$xSy = 1-(1-x)(1-y) = 1-(1-x-y+xy)$$

Hence,

$$xSy = x+y-xy$$

(ii)
$$\begin{aligned} xSy &= 1-\max(0,(1-x)+(1-y)-1) \\ &= 1-\max(0,\ 1-(x+y)) \end{aligned}$$

Hence,

$$\begin{aligned} xSy &= 1-0 \text{ for } x+y \geq 1 \\ &= 1-(1-(x+y)) \text{ for } x+y < 1 \end{aligned}$$

Hence,

$$\begin{aligned} xSy &= 1 \text{ for } x+y \geq 1 \\ &= x+y \text{ for } x+y < 1 \end{aligned}$$

or,

$$xSy = \min(1, x+y)$$

The T-norm represents a *generalized intersection* and the S-norm represents a *generalized union* of two fuzzy sets.