MEMC-5173
Intelligent Tools for Engineering Applications

Assignment #5 reference solution

Q. 1 (Q 5.2 Book 1)

$$\frac{\partial f}{\partial x} = \alpha \sec h^2(\alpha x) = \alpha[1-\tanh^2(\alpha x)] = \alpha[1-\tanh(\alpha x)][1+\tanh(\alpha x)] = \alpha[1-f(x)][1+f(x)]$$

The back-propagation training algorithm for this structure is:

Step 1: Initialize weights and thresholds to small random values

Step 2: Apply input pattern

$$o_0 = x_1$$
$$o_1 = x_2$$
$$o_2 = x_3 = -1$$

Step 3: Forward propagation

$$O_3 = f(w_{30}o_0+w_{31}o_1+w_{32}o_2) = f(w_{30}o_0+w_{31}o_1-w_{32})$$
$$O_4 = f(w_{40}o_0+w_{41}o_1+w_{42}o_2) = f(w_{40}o_0+w_{41}o_1-w_{42})$$
$$O_5 = -1$$
$$O_6 = f(w_{63}o_3+w_{64}o_4+w_{65}o_5) = f(w_{63}o_3+w_{64}o_4-w_{65})$$
$$O_7 = f(w_{73}o_3+w_{74}o_4+w_{75}o_5) = f(w_{73}o_3+w_{74}o_4-w_{75})$$
$$O_8 = -1$$
$$O_9 = f(w_{96}o_6+w_{97}o_7+w_{98}o_8) = f(w_{96}o_6+w_{97}o_4-w_{98})$$

Step 4: Output error measure

$$E = \tfrac{1}{2}(t-o_9)^2 + E$$
$$\delta_9 = f'(tot_9)(t-o_9)$$
$$= \alpha(1-o_9)(1+o_9)(t-o_9)$$

Step 5: Error back-propagation

Fourth layer weight updates:

$$\Delta w_{96} = \eta\delta_9 o_6 \quad w_{96}^{new} = w_{96}^{old} + \Delta w_{96}$$
$$\Delta w_{97} = \eta\delta_9 o_7 \quad w_{97}^{new} = w_{97}^{old} + \Delta w_{97}$$
$$\Delta w_{98} = \eta\delta_9 o_8 \quad w_{98}^{new} = w_{98}^{old} + \Delta w_{98}$$

Third layer error signals:

$$\delta_6 = f'(tot_6)\sum_{i=9}^{9} w_{i6}\delta_i = \alpha(1-o_6)(1+o_6)w_{96}\delta_9$$
$$\delta_7 = f'(tot_7)\sum_{i=9}^{9} w_{i7}\delta_i = \alpha(1-o_7)(1+o_7)w_{97}\delta_9$$

Third layer weight updates:

$$\Delta w_{63} = \eta\delta_6 o_3 \quad w_{63}^{new} = w_{63}^{old} + \Delta w_{63}$$
$$\Delta w_{64} = \eta\delta_6 o_4 \quad w_{64}^{new} = w_{64}^{old} + \Delta w_{64}$$
$$\Delta w_{65} = \eta\delta_6 o_5 \quad w_{65}^{new} = w_{65}^{old} + \Delta w_{65}$$

$$\Delta w_{73} = \eta\delta_7 o_3 \quad w_{73}^{new} = w_{73}^{old} + \Delta w_{73}$$
$$\Delta w_{74} = \eta\delta_7 o_4 \quad w_{74}^{new} = w_{74}^{old} + \Delta w_{74}$$
$$\Delta w_{75} = \eta\delta_7 o_5 \quad w_{75}^{new} = w_{75}^{old} + \Delta w_{75}$$

Second layer error signals:

$$\delta_3 = f'(tot_3)\sum_{i=6}^{7} w_{i3}\delta_i = \alpha(1-o_3)(1+o_3)\left(w_{63}\delta_6 + w_{73}\delta_7\right)$$
$$\delta_4 = f'(tot_4)\sum_{i=6}^{7} w_{i4}\delta_i = \alpha(1-o_4)(1+o_4)\left(w_{64}\delta_6 + w_{74}\delta_7\right)$$

Second layer weight updates:

$$\Delta w_{30} = \eta\delta_3 o_0 \quad w_{30}^{new} = w_{30}^{old} + \Delta w_{30}$$
$$\Delta w_{31} = \eta\delta_3 o_1 \quad w_{31}^{new} = w_{31}^{old} + \Delta w_{31}$$
$$\Delta w_{32} = \eta\delta_3 o_2 \quad w_{32}^{new} = w_{32}^{old} + \Delta w_{32}$$

$$\Delta w_{40} = \eta\delta_4 o_0 \quad w_{40}^{new} = w_{40}^{old} + \Delta w_{40}$$
$$\Delta w_{41} = \eta\delta_4 o_1 \quad w_{41}^{new} = w_{41}^{old} + \Delta w_{41}$$
$$\Delta w_{42} = \eta\delta_4 o_2 \quad w_{42}^{new} = w_{42}^{old} + \Delta w_{42}$$

Step 6: Repeat the process starting from step 2 using another exemplar. Once all exemplars have been used, we finish one epoch.

Step 7: Check if the cumulative error in the output layer is acceptable. If so the network has been trained. If not, repeat the whole process until the desired cumulative error is achieved.

Q. 2 (Q 12.2 Book 2)

Node functions in layer 4 need to be change to reflect the equations $f_1 = r_1$ and $f_2 = r_2$. The resulting ANFIS has 14 parameters, including 12 premise and 2 consequent parameters.

Q. 3 (Q 12.5 Book 2)

Let

$$\begin{aligned} g_1(x) &= k_1 \exp\left[-\left(\frac{x-c_1}{a_1}\right)^2\right], \\ g_2(x) &= k_2 \exp\left[-\left(\frac{x-c_2}{a_2}\right)^2\right]. \end{aligned}$$

Then

$$\begin{aligned} g_1(x)g_2(x) &= k_1 \exp\left[-\left(\tfrac{x-c_1}{a_1}\right)^2\right] k_2 \exp\left[-\left(\tfrac{x-c_2}{a_2}\right)^2\right] \\ &= k_1k_2 \exp\left[-\left(\tfrac{x-c_1}{a_1}\right)^2 - \left(\tfrac{x-c_2}{a_2}\right)^2\right] \\ &= k_1k_2 \exp\left[-\left(\frac{x - \frac{c_1a_2^2 + c_2a_1^2}{a_1^2+a_2^2}}{\frac{a_1a_2}{\sqrt{a_1^2+a_2^2}}}\right)^2 - \frac{(c_1-c_2)^2}{a_1^2+a_2^2}\right] \\ &= k_1k_2 \exp\left[-\frac{(c_1-c_2)^2}{a_1^2+a_2^2}\right] \exp\left[-\left(\frac{x - \frac{c_1a_2^2 + c_2a_1^2}{a_1^2+a_2^2}}{\frac{a_1a_2}{\sqrt{a_1^2+a_2^2}}}\right)^2\right] \end{aligned}$$

Q. 4 (Q 12.6 Book 2)

(a) $c_{i+1} - c_i = |a_{i+1}| + |a_i|$, $i = 1, 2, 3$.

(b) $c_{i+1} - c_i = \left(\frac{7}{3}\right)^{\frac{1}{2b}} (|a_{i+1}| + |a_i|)$, $i = 1, 2, 3$.

(c) $|a_{i+1}| + |a_i| \le c_{i+1} - c_i \le \left(\frac{7}{3}\right)^{\frac{1}{2b}} (|a_{i+1}| + |a_i|)$, $i = 1, 2, 3$.