EMEC-2434

Reference Solution of Assignment #1

Q1: Compute the Laplace transform (LT) using the LT properties and transform pairs.

(a) $x(t) = 1 + 3t + e^{2t}$

$$X(s) = \frac{1}{s} + \frac{3}{s^2} + \frac{1}{s-2}$$

(b) $x(t) = 2e^{-t}\sin(3t)$

$$X(s) = 2\frac{3}{(s+1)^2 + 3^2} = \frac{6}{s^2 + 2s + 10}$$

(c) $f(t) = \frac{d^2 x(t)}{d^2 t} + 2\frac{dx(t)}{dt}$

Assume: $f(t) \leftrightarrow F(s)$, $x(t) \leftrightarrow X(s)$

$F(s) = s^2 X(s) - sx(0) - \dot{x}(0) + 2[sX(s) - x(0)]$

With initial conditions: $\dot{x}(0) = 0$, $x(0) = 0$

$F(s) = s^2 X(s) + 2sX(s)$

(d) $v(t) = 2\int x(t)dt$

Assume: $v(t) \leftrightarrow V(s)$, $x(t) \leftrightarrow X(s)$

$V(s) = 2\frac{1}{s}X(s) = \frac{2}{s}X(s)$

Q2. How many measurements in a data set subject to random errors lie outside deviation $\pm 2\sigma$ and $\pm 3\sigma$, respectively.

(a) Probability outside deviation $\pm 2\sigma$

$P(d < -2\sigma) + P(d \geq 2\sigma)$, with normalization and let $z = \frac{d}{\sigma}$

$P\left(\frac{d}{\sigma} < -2\right) + P\left(\frac{d}{\sigma} \geq 2\right) = P(z < -2) + P(z \geq 2)$

$= (1 - P(z < 2)) + (1 - P(z < 2))$

= (1 - 0.9772) + (1 - 0.9772) = 0.0456 = 4.5%

(b) Probability outside deviation $\pm 3\sigma$

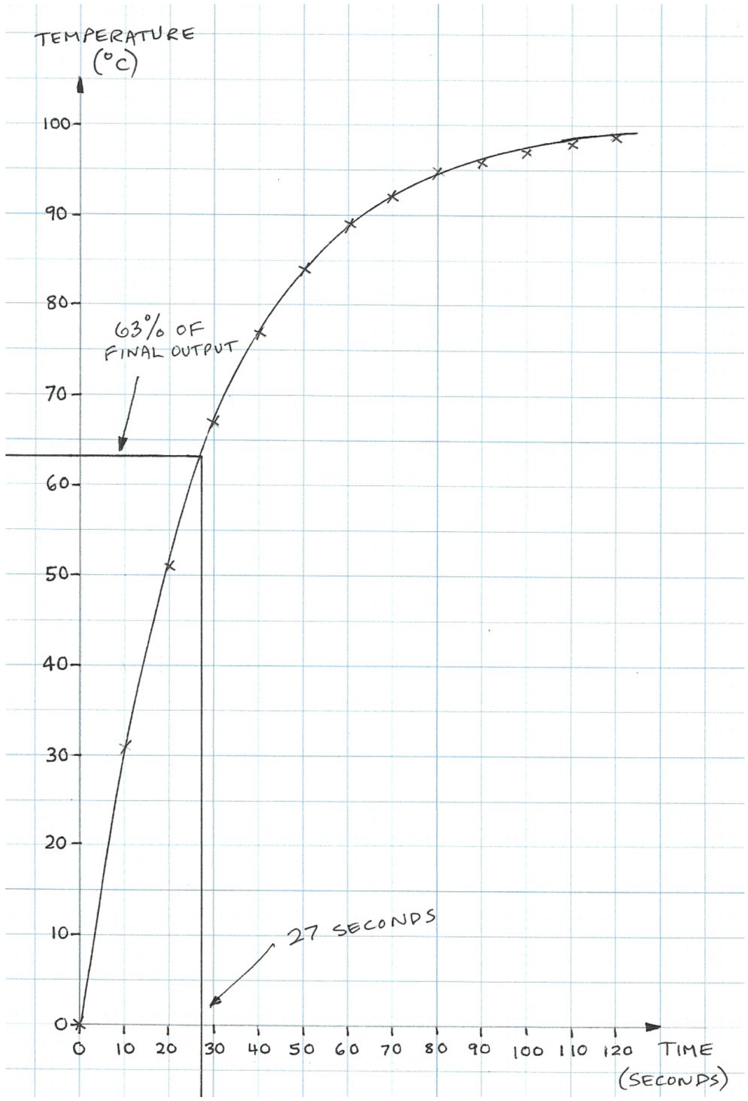
$P(d < -3\sigma) + P(d \geq 3\sigma)$, with normalization and let $z = \frac{d}{\sigma}$

$P\left(\frac{d}{\sigma} < -3\right) + P\left(\frac{d}{\sigma} \geq 3\right) = P(z < -3) + P(z \geq 3)$

$= (1 - P(z < 3)) + (1 - P(z < 3))$

= (1 - 0.9986) + (1 - 0.9986) = 0.0028 = 0.28%

Q. 3



The time constant is the time taken for the output reading to rise to 63% of its final value. Since the output reading is rising from 0ºC to 100ºC, this means the time when the output has risen to 63 ºC. Using the graph of temperature readings, this point is reached after 27 seconds. Thus the time constant of the thermometer is 27 seconds.

Q. 4

Part (a) The 25 measurements are: 9.4 10.1 9.1 12.3 10.3 10.0 10.5 9.0 10.8 10.0 11.1 9.8 7.6 9.2 10.7 8.4 11.0 9.7 11.3 8.7 9.9 11.5 10.0 9.5 11.9

The following data is given: Mean (μ) = 10.07 ; standard deviation (σ) = 1.11

The proportion of measurements < 11.05 can be expressed as: $P(x < 11.05)$

Rewriting this in terms of the z variable where $z = (x - \mu)/\sigma$:

For $x = 11.05, z = 0.8828$. Hence, $P(x < 11.05) = F(0.8828) = 0.8123$ (using standard z-function table).

Hence 81.23% of measurements are < 11.05. Since the total number of measurements is 25, 81.23% equates to 20.3. Since the number of measurements has to be an integer number, this means that the calculated number of measurements < 11.05 is 20. [30%]

Part (b) The proportion of measurements >9.55 be expressed as: $P(x > 9.55)$

For $x = 9.55, z = -0.468$

$P(x > 9.55) = 1 - P(x < 9.55) = 1 - F(-0.468) = 1 - [1 - F(0.468)] = F(0.468) = 0.6801$ (using standard z-function table).

Hence 68.1% of measurements are > 9.55. Since the total number of measurements is 25, 68.1% equates to 17.0. Thus, the best estimate is that 17 measurements are >9.55.

Part (c) The proportion of measurements (N) between 9.95 and 10.95 can be expressed as:

$N = P(x < 10.95) - P(x < 9.95)$

For $x = 10.95, z = +0.793$ and for $x = 9.95, z = -0.108$

Thus, N can be expressed as:

$$N = F(0.793) - F(-0.108) = F(0.793) - [1 - F(0.108)] = F(0.793) + F(0.108) - 1 = 0.7861 + 0.5430 - 1 = 0.329$$

32.9% of 25 = 8.23 measurements lie between 9.95 and 10.95. Rounding this result to the nearest integer, the best estimate is that 8 measurements lie between 9.95 and 10.95.

To check results, write out data values in ascending order:
7.6 8.4 8.7 9.0 9.1 9.2 9.4 9.5 9.7 9.8 9.9 10.0 10.0 10.0 10.1 10.3 10.5 10.7 10.8 11.0 11.1 11.3 11.5 11.9 12.3
Counting the number of measurements in each range gives:
measurements < 11.05 = 20
measurements > 9.55 = 17
measurements between 9.95 and 10.95 = 8
These counts agree with the calculated results.

Q. 5

Given data: 25.5 31.1 29.6 32.4 39.4 28.9 33.3 31.4 29.5 30.5 31.7 29.2

Number of measurements (n) = 12 ; mean = 31.04

Using x to represent data values, d to represent each deviation ($x - x_{mean}$), the deviations and deviations^2 are calculated in the following table:

x	25.5	31.1	29.6	32.4	39.4	28.9	33.3	31.4	29.5	30.5	31.7	29.2
d	-5.54	0.06	-1.44	1.36	8.36	-2.14	2.26	0.36	-1.54	-0.54	0.66	-1.84
d^2	30.71	0.00	2.08	1.85	69.86	4.59	5.10	0.13	2.38	0.29	0.43	3.39

Now forming sum of deviations^2, $\sum\left(deviation^2\right) = 120.81$

Hence, $\sigma = \sqrt{\frac{\sum\left(deviations^2\right)}{n-1}} = \sqrt{\frac{120.81}{11}} = 3.31$

Standard error of the mean $\alpha = \sigma/\sqrt{n} = 3.31/\sqrt{12} = 0.96$

Error range to 95.4% confidence level = ±2α = 1.92

Thus the mean petrol consumption to a 95.4% confidence level should be expressed as 31.0 ±1.9 (the original data is only given to an accuracy of one figure after the decimal point and it would be meaningless to try and express the calculated mean value and error range to any greater accuracy)

Q. 6

Part (i)

Given data: μ = mean temperature = 75°F ; σ = standard deviation = 2.15

Applying $z = (x-\mu)/\sigma$; For $x = 70,\ z = -2.3256$

$P[x \le 70] = 1 - P[x \ge 70] = 1 - P[z \ge -2.3256] = P[z \le -2.3256] = F(-2.3256) = 1 - F(+2.3256)$

Using error function (z-function or gaussian) table:

$1 - F(2.3256) = 1 - 0.99 = 0.1$

Thus, the temperature is less than 70°F for 1% of the time.

Part (ii)

Probability that temperature (x) is between 73°F and 77°F is given by:

$P[x < 77] - P[x < 73]$

Applying $z = (x-\mu)/\sigma$; For $x = 73,\ z = -0.9302,\ F(z) = 0.1762$

For $x = 77,\ z = +0.9302,\ F(z) = 0.8238$

$P[x < 77] - P[x < 73] = P[z < 0.9302] - P[z < -0.9302] = 0.8238 - 0.1762 = 0.6476$

Thus, the temperature between 73°F and 77°F for 65% of the time (rounding from 64.76%).