EMTR 2017: Robotics and Automation I
Assignment 1
Reference Solutions

Question 1-4

Non-servo robots: Materials handling, servicing a special purpose machine such as a press.
Point-to-point robots: Materials handling, spot welding, forging.
Continuous path: Arc welding, grinding and deburring, spray painting, assembly, sheep shearing.

Question 2-5

a) For any matrices A and B, $\det(A^T) = \det(A)$ and $\det(AB) = \det(A)\det(B)$. Thus, if R is orthogonal

$$1 = \det(I) = \det(R^T R) = \det(R^T)\det(R) = (R)^2$$

which implies that

$$\det R = \pm 1.$$

b) For a right-handed coordinate system, $r_1 \times r_2 = r_3$. This implies that

$$r_{12}r_{23} - r_{13}r_{22} = r_{31}; \quad -r_{11}r_{23} + r_{13}r_{21} = r_{32}; \quad r_{11}r_{22} - r_{12}r_{21} = r_{33}.$$

Therefore, expanding $\det R$ about column 3 gives

$$\begin{aligned}
\det R &= \det \begin{bmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{bmatrix} \\
&= r_{31}(r_{12}r_{23} - r_{22}r_{13}) - r_{32}(r_{11}r_{23} - r_{21}r_{13}) + r_{33}(r_{11}r_{22} - r_{21}r_{12}) \\
&= r_{31}(r_{31}) + r_{32}(r_{32}) + r_{33}(r_{33}) \\
&= \|r_3\|^2 = 1.
\end{aligned}$$

Question 2-8

For a rotation of θ about the x axis we have

$$\begin{aligned}
x_0 \cdot x_1 &= 1 \\
y_0 \cdot y_1 &= \cos\theta \\
z_0 \cdot z_1 &= \cos\theta \\
z_0 \cdot y_1 &= \sin\theta \\
y_0 \cdot z_1 &= -\sin\theta
\end{aligned}$$

and all other dot products are zero. Substituting into the rotation matrix in Section 2.2.2 gives

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}.$$

For a rotation of θ about the y axis we have

$$\begin{aligned} y_0 \cdot y_1 &= 1 \\ x_0 \cdot x_1 &= \cos\theta \\ z_0 \cdot z_i &= \cos\theta \\ z_0 \cdot x_1 &= -\sin\theta \\ x_0 \cdot z_1 &= \sin\theta \end{aligned}$$

and all other dot products are zero. Again using the rotation matrix gives

$$R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}.$$

Verify they also satisfy properties analogous to Eq. (2.4)-(2.6).

If $\theta = 0$, $R_{x,0} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}_{\theta=0} = I$

$$R_{x,\theta}R_{x,\phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta\cos\phi - \sin\theta\sin\phi & -\cos\theta\sin\phi - sin\theta\cos\phi \\ 0 & \sin\theta\cos\phi + \cos\theta\sin\phi & -sin\theta\sin\phi + \cos\theta\cos\phi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta+\phi) & -\sin(\theta+\phi) \\ 0 & sin(\theta+\phi) & \cos(\theta+\phi) \end{bmatrix} = R_{x,\theta+\phi}$$

$$(R_{x,\theta})^{-1} = (R_{x,\theta})^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$R_{x,-\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\theta) & -\sin(-\theta) \\ 0 & \sin(-\theta) & \cos(-\theta) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix} = (R_{x,\theta})^{-1}$$

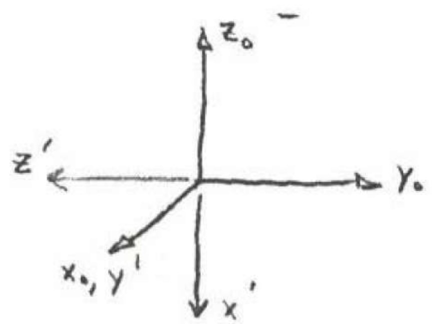
Similarly, you can verify $R_{y,0} = I$, $R_{y,\theta}R_{y,\phi} = R_{y,\theta+\phi}$, $(R_{y,\theta})^{-1} = R_{y,-\theta}$

Question 2-12

$$R = R_{z,\alpha}R_{x,\phi}R_{z,\theta}R_{x,\psi}$$

Question 2-14

$$R \;=\; R_{y,\frac{\pi}{2}} R_{x,\frac{\pi}{2}} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$



Question 2-15

$$R_3^2 = R_1^2 R_3^1 \quad \text{where} \quad R_1^2 = (R_2^1)^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & 1/2 \end{bmatrix}.$$

Therefore,

$$R_3^2 \;=\; \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & \sqrt{3}/2 \\ 0 & -\sqrt{3}/2 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ \sqrt{3}/2 & 1/2 & 0 \\ 1/2 & -\sqrt{3}/2 & 0 \end{bmatrix}$$