EMTR 2017: Robotics and Automation I
Assignment 2 Reference Solution

## Problem 2-22

Compute the rotation matrix given by the product

$$R_{x,\theta}R_{y,\phi}R_{z,\pi}R_{y,-\phi}R_{x,-\theta}$$

$R_{x,\theta}R_{y,\phi}R_{z,\pi}R_{y,-\phi}R_{x,-\theta}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \cos(\phi) & 0 & \sin(\phi) \\ 0 & 1 & 0 \\ -\sin(\phi) & 0 & \cos(\phi) \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\phi) & 0 & -\sin(\phi) \\ 0 & 1 & 0 \\ \sin(\phi) & 0 & \cos(\phi) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$= \begin{bmatrix} -\cos(2\phi) & -2\cos(\phi)\sin(\phi)\sin(\theta) & \cos(\theta)\sin(2\phi) \\ -2\cos(\phi)\sin(\phi)\sin(\theta) & -\cos(\theta)^2 - \cos(2\phi)\sin(\theta)^2 & -\cos(\phi)^2\sin(2\theta) \\ \cos(\theta)\sin(2\phi) & -\cos(\phi)^2\sin(2\theta) & \cos(\phi)^2\cos(\theta)^2 - \cos(\theta)^2\sin(\phi)^2 - \sin(\theta)^2 \end{bmatrix}$$

## Problem 2-24

Find the rotation matrix corresponding to the set of Euler angles $\{\frac{\pi}{2}, 0, \frac{\pi}{4}\}$. What is the direction of the $x_1$ axis relative to the base frame?

$$R_1^0 = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

The direction of the $x$-axis is $\left(0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)^T$.

## Problem 2-36

Compute the homogeneous transformation representing a translation of 3 units along the $x$-axis followed by a rotation of $\frac{\pi}{2}$ about the current $z$-axis followed by a translation of 1 unit along the fixed $y$-axis. Sketch the frame. What are the coordinates of the origin $O_1$ with respect to the original frame in each case?

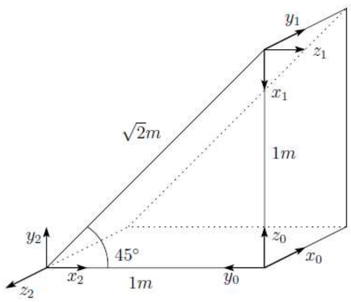
$$T = T_{y,1}T_{x,3}T_{z,\pi/2}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 3 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Problem 2-37

Consider the diagram of Figure 2.15. Find the homogeneous transformations $H_1^0, H_2^0, H_2^1$ representing the transformations among the three frames shown. Show that $H_2^0 = H_1^0, H_2^1$.




$$H_1^0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad H_2^0 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad H_2^1 = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Problem 3-1

Consider the three-link planar manipulator shown in Figure 3.23. Derive

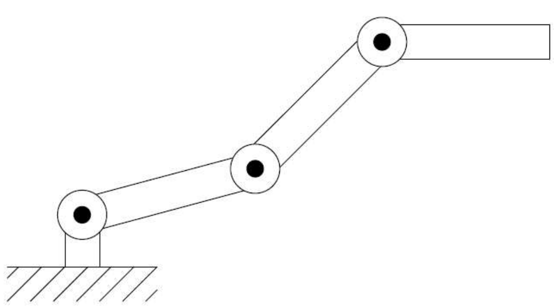
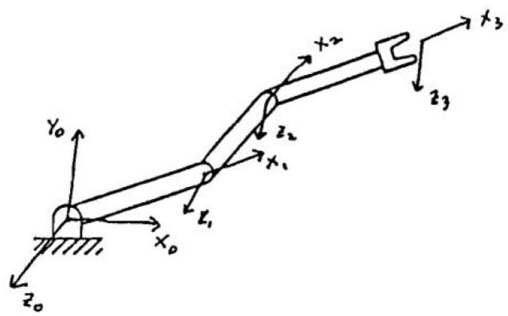


*Fig. 3.23* Three-link planar arm of Problem 3-2

the forward kinematic equations using the DH-convention.




| link | $a_1$ | $\alpha_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | $\alpha_1$ | 0 | 0 | $\theta_1$ |
| 2 | $\alpha_2$ | 0 | 0 | $\theta_2$ |
| 3 | $\alpha_3$ | 0 | 0 | $\theta_3$ |

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; A_2 = \begin{bmatrix} c_2 & -c_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1A_2A_3 = \begin{bmatrix} c_{123} & -s_{123} & 0 & a_1c_1 + a_2c_{12} + a_3c_{123} \\ s_{123} & c_{123} & 0 & a_1s_1 + a_2s_{12} + a_3s_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Problem 3-5

Consider the three-link articulated robot of Figure 3.27. Derive the for-

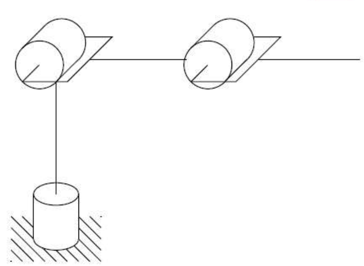
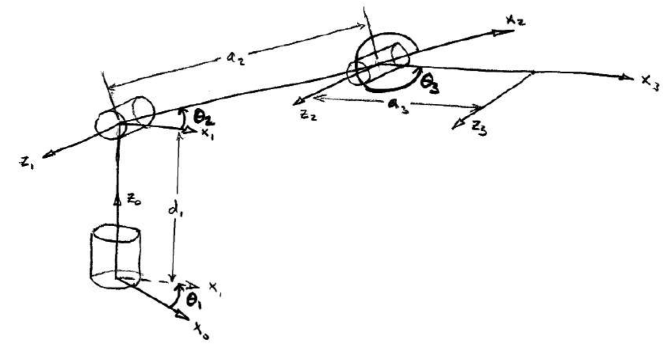


*Fig. 3.27* Three-link articulated robot

ward kinematic equations using the DH-convention.




| link | $a_i$ | $\alpha_i$ | $d_i$ | $\theta_i$ |
|---|---|---|---|---|
| 1 | 0 | 90 | 0 | $\theta_1$ |
| 2 | $a_2$ | 0 | 0 | $\theta_2$ |
| 3 | $a_3$ | 0 | 0 | $\theta_3$ |

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$$\begin{aligned}
r_{11} &= c_1c_2c_3 - c_1s_2s_3 = c_1c_{23} \\
r_{12} &= -c_1c_2s_3 - c_1c_3c_2 = -c_1s_{23} \\
r_{13} &= s_1 \\
d_x &= a_2a_2c_1c_2 + a_3c_1c_2c_3 - a_3c_1s_2s_3 = a_2c_1c_2 + a_3c_1c_{23} \\
r_{21} &= c_2c_3s_1 - s_1s_2s_3 = x_1c_{23} \\
r_{22} &= -c_2s_1s_3 - c_3s_1s_2 = -s_1s_{23} \\
r_{23} &= -c_1 \\
d_y &= a_2c_2s_1 + a_3c_2c_3s_1 - a_3s_1s_2s_3 = a_2c_2s_1 + a_3s_1c_{23} \\
r_{31} &= c_2s_3 + c_3s_2 = s_{23} \\
r_{32} &= c_2c_3 - s_2s_3 = c_{23} \\
r_{33} &= 0 \\
d_z &= a_2s_2 + a_3c_2s_3 + a_3c_3s_2 = a_2s_2 + a_3s_{23}
\end{aligned}$$