EMTR 2017: Robotics and Automation I

Assignment 3: **Reference** Solutions

Problem 3-6

Consider the three-link cartesian manipulator of Figure 3.28. Derive the forward kinematic equations using the DH-convention.

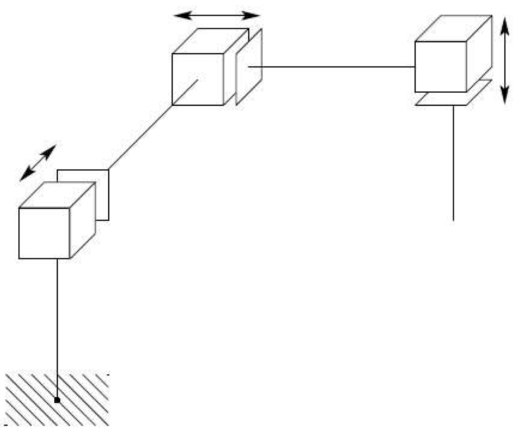
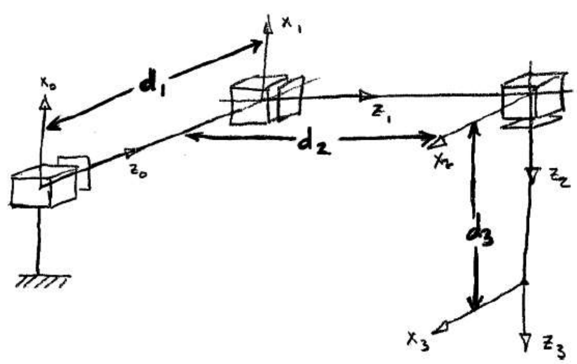


Fig. 3.28 Three-link cartesian robot



link	a_i	α_i	d_i	θ_i
1	0	$-90°$	d_1	0
2	0	$90°$	d_2	$90°$
3	0	0	d_3	$-90°$

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^3 = A_1A_2A_3 = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ -1 & 0 & 0 & d_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3-7

Attach a spherical wrist to the three-link articulated manipulator of Problem 3-6. as shown in Figure 3.29. Derive the forward kinematic equations for this manipulator.

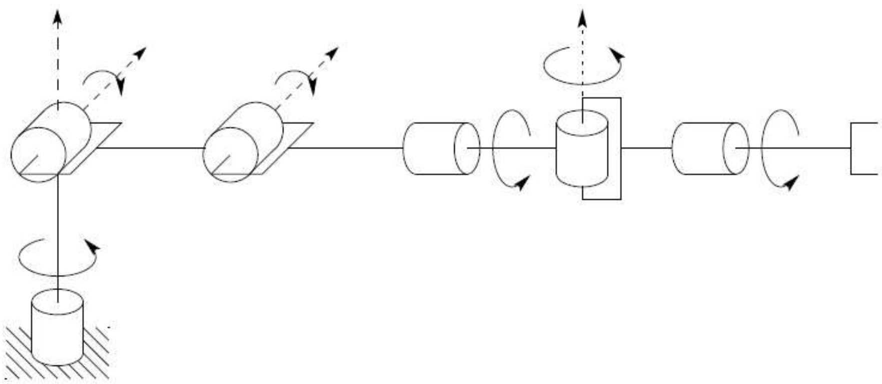


Fig. 3.29 Elbow manipulator with spherical wrist

link	a_i	α_i	d_i	θ_i
1	0	90°	0	θ_1
2	a_2	0	0	θ_2
3	a_3	0	0	θ_3
4	0	−90°	0	θ_4
5	0	0	0	θ_5
6	0	0	d_6	θ_6

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3c_3 \\ s_3 & c_3 & 0 & a_3s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^6 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{23} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
r_{11} &= c_1[c_5c_6c_{234} - s_6s_{234}] - s_1s_5c_6 \\
r_{12} &= -c_1[c_5s_6c_{234} + c_6s_{234}] + s_1s_5s_6 \\
r_{13} &= c_1s_5c_{234} + s_1c_5 \\
d_x &= a_2c_1c_2 + a_3c_1c_{23} + d_6[c_1s_5c_{234} + s_1c_5] \\
r_{21} &= c_1s_5s_6 + s_1c_5c_6c_{234} - s_1s_6s_{234} \\
r_{22} &= -c_1s_5s_6 - s_1c_5s_6c_{234} \\
r_{23} &= -c_1c_5 + s_1s_5c_{234} \\
d_y &= a_2s_1c_2 + a_3s_1c_{23} - d_6[c_1c_5 + s_1s_5c_{234}] \\
r_{31} &= s_6c_{234} + c_5s_6s_{234} \\
r_{32} &= c_6s_{234} - c_5s_6s_{234} \\
r_{33} &= s_5s_{234} \\
d_z &= a_2s_2 + a_3c_2s_{23} + d_6s_5s_{234}
\end{aligned}$$

Problem 3-8

Attach a spherical wrist to the three-link cartesian manipulator of Problem 3-7 as shown in Figure 3.30. Derive the forward kinematic equations for this manipulator.

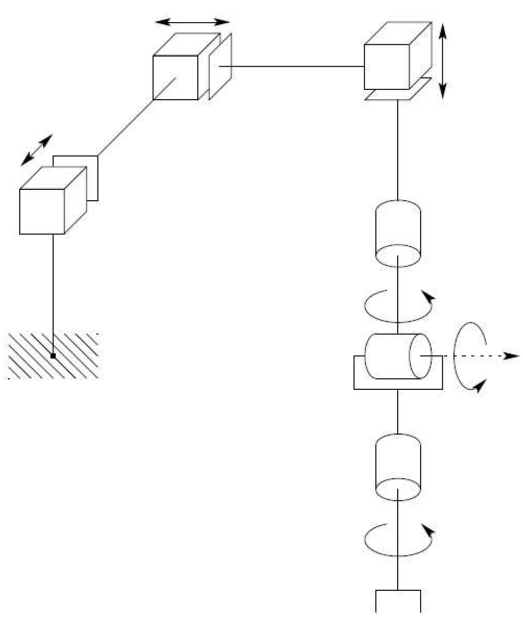


Fig. 3.30 Cartesian manipulator with spherical wrist

Attaching a spherical wrist to the robot of Problem 3-7 gives

$$T_0^5 = T_0^3 T_3^6$$

The matrix T_0^3 is given as in Problem 3-7. The matrix T_3^6 is given by Equation (3.15) of the text. Therefore

$$T_0^6 = \begin{bmatrix} -c_6 s_5 & s_5 s_6 & c_5 & d_3 + d_6 c_5 \\ -c_4 c_5 c_6 + s_4 s_6 & c_4 c_5 s_6 + c_6 s_4 & -c_4 s_5 & d_2 - d_6 c_4 s_5 \\ -c_4 s_6 - c_5 c_6 s_4 & -c_4 c_6 + c_5 s_4 s_6 & -s_4 s_5 & d_1 - d_6 s_4 s_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 4-1

Verify Equation (4.6) by direct calculation.

It is a very simple question. Solve this question as what we did in the examples in the lecture.

Problem 4-4

Verify Equation (4.16) by direct calculation.

$$R_{x,90} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}; S(a) = \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix}; Ra = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$$

$$S(Ra) = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

Then

$$RS(a)R^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix} = S(Ra)$$

Problem 4-6

Given $R = R_{x,\theta} R_{y,\phi}$, compute $\frac{\partial R}{\partial \phi}$. Evaluate $\frac{\partial R}{\partial \phi}$ at $\theta = \frac{\pi}{2}$, $\phi = \frac{\pi}{2}$.

$$R_1^0 = R_{x,\theta} R_{y,\phi}$$

Then

$$\frac{\partial R_0^1}{\partial \phi} = R_{x,\theta} \frac{\partial R_{y,\phi}}{\partial \phi} = R_{x,\theta} S(j) R_{y,\phi} = \begin{bmatrix} -s\phi & 0 & c\phi \\ s\theta c\phi & 0 & s\theta s\phi \\ -c\theta c\phi & 0 & -s\phi c\theta \end{bmatrix}$$

$$\left.\frac{\partial R_0^1}{\partial \phi}\right|_{\substack{\theta=\phi/2 \\ \phi=\pi/2}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Problem 4-10

Two frames $o_0x_0y_0z_0$ and $o_1x_1y_1z_1$ are related by the homogeneous transformation

$$H = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

A particle has velocity $v_1(t) = (3, 1, 0)^T$ relative to frame $o_1x_1y_1z_1$. What is the velocity of the particle in frame $o_0x_0y_0z_0$?

$$\begin{aligned} p_0 &= Rp_1 + d \\ \dot{p}_0 &= R\dot{p}_1 \\ &= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} \end{aligned}$$